

# The Rise of the Proton Structure Function $F_2$ Towards Low $x$ \*

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Results on the derivative of  $\log(F_2)$  with respect to  $\log(x)$  at fixed  $Q^2$  are presented. The measured derivatives are within errors independent of  $x$  for  $Q^2 \geq 0.85 \text{ GeV}^2$  and increase linearly with  $\log(Q^2)$  for  $10^{-4} \leq x \leq 0.01$  and  $Q^2 \gtrsim 3 \text{ GeV}^2$ . The results are based on preliminary and published H1 data which at  $Q^2$  below  $2 \text{ GeV}^2$  are combined with NMC and ZEUS data.

## 1. Introduction

The rise of the proton structure function  $F_2$  towards small Bjorken  $x$  has been discussed since the existence of QCD. In the double asymptotic limit (large energies, i.e. small  $x$ , and large photon virtualities  $Q^2$ ) the DGLAP evolution equations [1] can be solved [2] and  $F_2$  is expected to rise approximately like a power of  $x$  towards low  $x$ . A power like behaviour is also expected in the BFKL approach [3]. However, it soon was discussed [4] that this rise may eventually be limited by gluon self interactions in the nucleon, or more generally due to unitarity constraints.

Experimentally this rise towards small  $x$  was first observed in 1993 in the HERA data [5]. Meanwhile the precision of the  $F_2$  data is much improved and the rise can be studied in great detail.

## 2. Procedure

The low  $x$  behaviour of  $F_2$  at fixed  $Q^2$  is studied locally by the measurement of the derivative  $\lambda \equiv -(\partial \ln F_2 / \partial \ln x)_{Q^2}$  as function of  $x$  and  $Q^2$ . The results are based on preliminary H1  $F_2$  data presented to this conference [6] covering the range  $0.5 < Q^2 < 3.5 \text{ GeV}^2$  and published H1 data [7], [8] which cover the range  $1.5 < Q^2 < 150 \text{ GeV}^2$ . The low  $Q^2$   $F_2$  data were obtained by shifting the  $ep$  interaction vertex by 70 cm in proton beam direction [6]. At  $Q^2 < 2 \text{ GeV}^2$  the H1 data are also shown combined with

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data of NMC [9] and ZEUS [10]. The derivative  $\lambda(x, Q^2)$  is evaluated using data points at adjacent values of  $x$  at fixed  $Q^2$  taking into account error correlations and  $x$  spacing corrections. The derivatives are compared with the next to leading order (NLO) QCD fit to the H1 cross section data [7] and a “fractal” fit [11] where self-similar properties of the proton structure are assumed.

### 3. Results

The  $x$  and  $Q^2$  dependence of  $\lambda = -(\partial \ln F_2 / \partial \ln x)_{Q^2}$  is shown in Fig. 1.

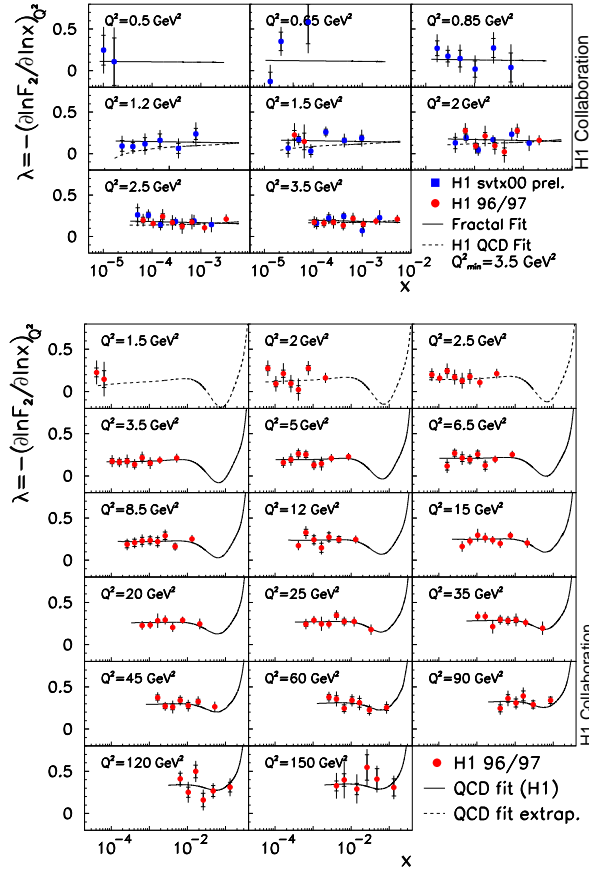


Fig. 1. Derivative  $\lambda = -(\partial \ln F_2 / \partial \ln x)_{Q^2}$  compared with the QCD analysis of ref. [7] and a “fractal” fit [11] for  $0.5 < Q^2 < 3.5 \text{ GeV}^2$  (upper plot) and for  $1.5 < Q^2 < 150 \text{ GeV}^2$  (lower plot)

The new shifted vertex and the published data agree well in the overlap region. The derivative  $\lambda$  is constant within experimental uncertainties for fixed  $Q^2$  in the range  $x < 0.01$ , implying that the data are consistent with the power behaviour  $F_2 = c(Q^2) \cdot x^{-\lambda(Q^2)}$ . Fitting this form for each  $Q^2$  bin to the data at  $x < 0.01$ , results in the  $\lambda$  and  $c$  values presented in Fig. 2.

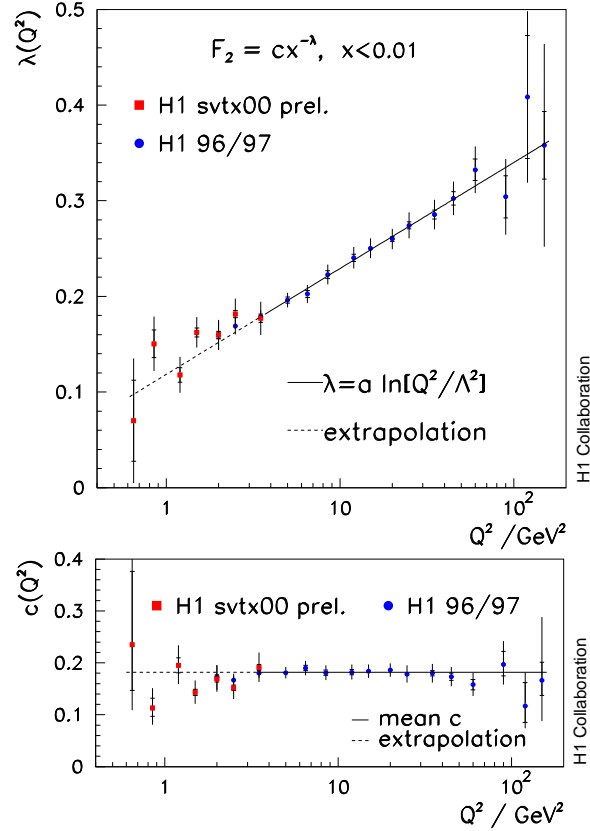


Fig. 2.  $\lambda(Q^2)$  and  $c(Q^2)$  from fits of the form  $F_2 = c(Q^2) \cdot x^{-\lambda(Q^2)}$  to the H1 structure function data [7] and [11].

The results show that the  $F_2$  data at low  $x$  for  $Q^2 \gtrsim 3.5 \text{ GeV}^2$  can be well described by the very simple parameterisation

$$F_2 = c \cdot x^{-\lambda(Q^2)}, \quad \text{with } \lambda(Q^2) = a \cdot \ln[Q^2/\Lambda^2] \quad (1)$$

with  $a = 0.0481 \pm .0013 \pm .0037$  and  $\Lambda = 292 \pm 20 \pm 51 \text{ MeV}$  and  $c \approx 0.18$ .

At low  $Q^2$  the deviation of  $\lambda$  from the logarithmic  $Q^2$  dependence and the decrease of  $c(Q^2)$  is more significant if the H1 data are combined with

NMC [9] and ZEUS [10] data (see Fig. 3).

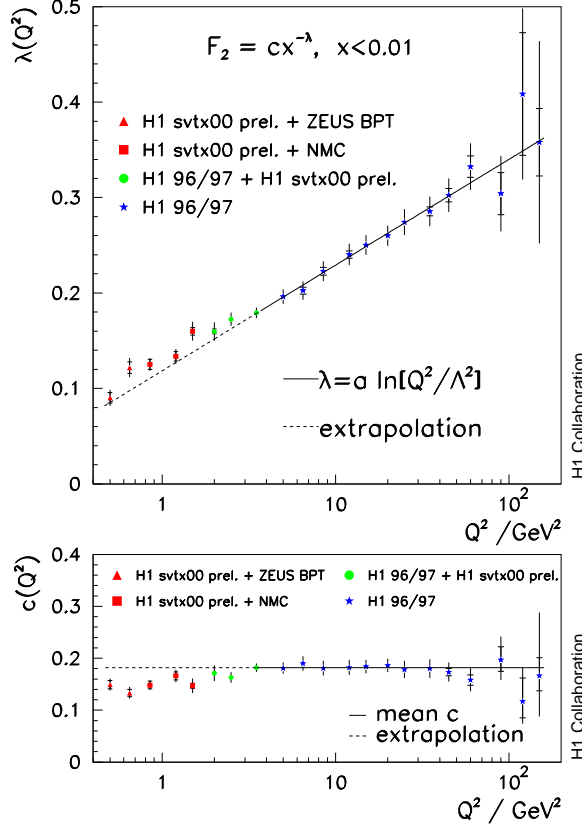


Fig. 3.  $\lambda(Q^2)$  and  $c(Q^2)$  from fits of the form  $F_2 = c(Q^2) \cdot x^{-\lambda(Q^2)}$  combining the H1 structure function data of [7] and [11] and the H1 data with data of NMC [9] and ZEUS [10].

The deviations from a simple constant respectively logarithmic behaviour occur at about such  $Q^2$  values below which perturbative QCD fits (e.g. [7]) are not supposed to be valid. At small  $Q^2$  the structure function  $F_2$  can be related to the total virtual photon absorption cross section by

$$\sigma_{tot}^{\gamma^*p} = 4\pi\alpha^2 F_2/Q^2 \sim x^{-\lambda}/Q^2 \quad (2)$$

where the total  $\gamma^*p$  energy squared is given by  $s = Q^2/x$ . For  $Q^2 \rightarrow 0$  we can expect  $c(Q^2) \rightarrow 0$  and  $\lambda(Q^2) \rightarrow \approx 0.08$ . The latter value corresponds to the energy dependence of soft hadronic interactions  $\sigma_{tot} \sim s^{\alpha_{\mathbb{P}}(0)-1}$  with  $\alpha_{\mathbb{P}}(0) - 1 \approx 0.08$  [12] which is approximately reached at  $Q^2 = 0.5 \text{ GeV}^2$ .

#### 4. Conclusion

No significant deviation from the power behaviour  $F_2 \sim x^{-\lambda}$  at fixed  $Q^2$  is visible at present energies and  $Q^2 \gtrsim 0.85 \text{ GeV}^2$ . More specifically:

- For  $x < 0.01$  the derivative  $\lambda \equiv -(\partial \ln F_2 / \partial \ln x)_{Q^2}$  is independent of  $x$  within errors.
- $\lambda$  is proportional to  $\ln(Q^2)$  for  $Q^2 \gtrsim 3 \text{ GeV}^2$ , i.e. in the pQCD region.
- Here the data can be very simply parametrised by  $F_2 = cx^{-\lambda(Q^2)}$ .
- At  $Q^2 \lesssim 3 \text{ GeV}^2$  deviations from the logarithmic  $Q^2$  dependence of  $\lambda$  are observed.
- At low  $Q^2$  ( $Q^2 \lesssim 1 \text{ GeV}^2$ ) the energy rise is similar as in soft hadronic interactions.

#### REFERENCES

- [1] Y. L. Dokshitzer, (In Russian),” Sov. Phys. JETP **46** (1977) 641 [Zh. Eksp. Teor. Fiz. **73** (1977) 1216];  
V. N. Gribov and L. N. Lipatov, Yad. Fiz. **15** (1972) 1218 [Sov. J. Nucl. Phys. **15** (1972) 675]; Yad. Fiz. **15** (1972) 781 [Sov. J. Nucl. Phys. **15** (1972) 438];  
G. Altarelli and G. Parisi, Nucl. Phys. B **126** (1977) 298.
- [2] A. De Rujula et al, Phys. Rev. D **10** (1974) 1649;  
R. D. Ball and S. Forte, Phys. Lett. B **335** (1994) 77.
- [3] E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP **44** (1976) 443 [Zh. Eksp. Teor. Fiz. **71** (1976) 840]; Sov. Phys. JETP **45** (1977) 199 [Zh. Eksp. Teor. Fiz. **72** (1977) 377];  
I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822 [Yad. Fiz. **28** (1978) 1597].
- [4] L. V. Gribov, E. M. Levin and M. G. Ryskin, Nucl. Phys. B **188** (1981) 555;  
Phys. Rept. **100** (1983) 1;  
A. H. Mueller and J. w. Qiu, Nucl. Phys. B **268** (1986) 427.
- [5] I. Abt *et al.* [H1 Collaboration], Nucl. Phys. B **407** (1993) 515;  
M. Derrick *et al.* [ZEUS Collaboration], Phys. Lett. B **316** (1993) 412.
- [6] T. Lastovicka, these proceedings.
- [7] C. Adloff *et al.* [H1 Collaboration], Eur. Phys. J. C **21** (2001) 33.
- [8] C. Adloff *et al.* [H1 Collaboration], Phys. Lett. B **520** (2001) 183.
- [9] M. Arneodo *et al.* [New Muon Collaboration.], Phys. Lett. B **364** (1995) 107;  
Nucl. Phys. B **483** (1997) 3.
- [10] J. Breitweg *et al.* [ZEUS Collaboration], Phys. Lett. B **487** (2000) 53.
- [11] T. Lastovicka, arXiv:hep-ph/0203260 and these proceedings.
- [12] A. Donnachie and P. V. Landshoff, Phys. Lett. B **296** (1992) 227.